

Five-quark picture of $\Lambda(1405)$ in anisotropic lattice QCD

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Five-quark (5Q) picture of $\Lambda(1405)$ is studied using quenched lattice QCD with an exotic 5Q operator of $N\bar{K}$ type. To discriminate mere $N\bar{K}$ and $\Sigma\pi$ scattering states, Hybrid Boundary Condition (HBC), a flavor-dependent boundary condition, is imposed on the quark fields along spatial direction. 5Q mass $m_{5Q} \simeq 1.89$ GeV is obtained after the chiral extrapolation to the physical quark mass region, which is too heavy to be identified with $\Lambda(1405)$. Then, $\Lambda(1405)$ seems neither a pure 3Q state nor a pure 5Q state, and therefore we present an interesting possibility that $\Lambda(1405)$ is a mixed state of 3Q and 5Q states.

$\Lambda(1405)$ is an $I = 0, S = -1, Q = 0$ negative-parity baryon. As is obvious from its quantum number, it contains a (heavy) strange quark as a valence quark. Nevertheless, $\Lambda(1405)$ is the lightest negative-parity baryon. (The lightest non-strange baryon is $N^*(1520)$.) In a quark-model interpretation, $\Lambda(1405)$ is identified as a flavor SU(3) singlet baryon. For this assignment, however, the anomalous value of the LS force is to be introduced. Then, one may wonder if there may be something behind, i.e., it may involve some exotic structure. Indeed, it has been conjectured, for a long period, that $\Lambda(1405)$ may be a bound state of N and \bar{K} via the strong interaction, i.e., a 5Q object rather than a 3Q one. If this is the case, its binding energy is $m_N + m_K - 1405 \simeq 30$ MeV, which seems to be natural magnitude of the binding energy for the hadronic molecule.

Baryon spectra in flavor SU(3) sector was studied by quenched lattice QCD.^{1), 2), 3), 4)} It was suggested that $\Lambda(1405)$ may have an exceptional feature. For instance, in Ref.[1], it was reported that baryon masses in flavor SU(3) sector can be reproduced by quenched lattice QCD within about 10 % deviation except for $\Lambda(1405)$, for which a significant overestimate of more than 300 MeV is reported. One of the most attractive explanations for this is provided by the 5Q picture of $\Lambda(1405)$, i.e., if $\Lambda(1405)$ is dominated by 5Q components, it should be difficult for quenched QCD with a conventional 3Q interpolating field to reproduce it.

In this paper, we will consider the 5Q picture of $\Lambda(1405)$ by making a constructive use of quenched QCD. We attempt to use the suppressed contributions from $q\bar{q}$ loops in quenched QCD in order to obtain an additional information on a possible exotic structure of $\Lambda(1405)$ in the following way. An ordinary 3Q interpolating field fully couples to 3Q intermediate states, whereas it couples to 5Q intermediate states only in an imperfect manner. On the other hand, a 5Q in-

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interpolating field can fully couple to 5Q intermediate states, whereas its couplings to 3Q intermediate states vanish at all, once we neglect the contribution from the annihilation diagram. We will use these quenched QCD features to study the nature of $\Lambda(1405)$ by comparing the mass spectrum obtained with an ordinary 3Q interpolating field and that with a 5Q interpolating field neglecting the annihilation diagram. For 3Q interpolating field, we employ a flavor-singlet interpolating field as $\Lambda^{(3Q)} \equiv \epsilon_{abc} [(u_a^T C \gamma_5 d_b) s_c + (d_a^T C \gamma_5 s_b) u_c + (s_a^T C \gamma_5 u_b) d_c]$, where a, b, c denote color indices, and $C \equiv \gamma_0 \gamma_2$ denotes the charge conjugation matrix. For 5Q interpolating field, we employ an iso-scalar 5Q interpolating field of $N\bar{K}$ type as $\Lambda^{(5Q)} \equiv pK^- - n\bar{K}^0$, where $p \equiv \epsilon_{abc} (u_a^T C \gamma_5 d_b) u_c$, $n \equiv \epsilon_{abc} (u_a^T C \gamma_5 d_b) d_c$, $K^- \equiv \bar{u}i\gamma_5 s$, and $\bar{K}^0 \equiv -\bar{d}i\gamma_5 s$.

5Q calculation involves an obstacle. Empirically, the mass gap between $\Lambda(1405)$ and the $N\bar{K}$ threshold is only about 30 MeV, which is too small for a practical lattice QCD calculation to identify these two states separately. To avoid this, we adopt a flavor-dependent boundary condition (BC) along the spatial directions (“*Hybrid Boundary Condition (HBC)*”),^{5),6)} which was proposed in the studies of $\Theta^+(1540)$. HBC consists of anti-periodic BC on u and d quark fields and periodic BC on s quark field. Since $\Lambda^{(5Q)}$ field contains even number of u and d fields, it is subject to the periodic BC, which allows us to consider the rest frame of the 5Q system $\Lambda(1405)$. In contrast, since p, n, K^-, \bar{K}^0 fields contain odd number of u and d fields, they are subject to the anti-periodic BC. Their spatial momenta are discretized as $\vec{p} = ((2n_x + 1)\pi/L, (2n_y + 1)\pi/L, (2n_z + 1)\pi/L)$, where $n_i \in \mathbb{Z}$, L is the spatial extension of the lattice. Note that $|\vec{p}|$ cannot vanish with HBC. Its minimum value is $p_{\min} \equiv \sqrt{3}\pi/L$, owing to which $N\bar{K}$ threshold is raised from $E_{\text{PBC,th}} \simeq m_N + m_K$ for PBC to $E_{\text{HBC,th}} \simeq \sqrt{m_N^2 + 3\pi^2/L^2} + \sqrt{m_K^2 + 3\pi^2/L^2}$ for HBC. For $L \simeq 2.2$ fm, the minimum value of $|\vec{p}|$ amounts to about 499 MeV leading to the shift in the threshold of more than 200 MeV, which make it possible to distinguish a possible $\Lambda(1405)$ state from $N\bar{K}$ threshold.

The 5Q calculation still involves a difficulty. $\Lambda(1405)$ is expected to be embedded in $\pi\Sigma$ “continuum” even with HBC. Note that $\Sigma\pi$ threshold is not raised by HBC. We have to distinguish $\Lambda(1405)$ as a compact 5Q state from $\pi\Sigma$ scattering states. If we were to impose such a spatial BC that the periodic BC is imposed on \bar{u} , \bar{d} , and \bar{s} fields, while the anti-periodic BC is imposed on u , d and s fields, then the both the $N\bar{K}$ and $\Sigma\pi$ threshold could be raised more than 200 MeV. However, this is problematic, since it does not respect the charge conjugation symmetry. Instead, we virtually introduce additional flavors u' and d' , and regard \bar{u} and \bar{d} in 5Q $\Lambda(1405)$ as anti-quarks for these two additional flavors, i.e., \bar{u}' and \bar{d}' , respectively. We emphasize that this will not change anything, because we neglect the annihilation diagram as is mentioned before. Now, we consider a modified HBC, which will be referred to as “*HBC2*”. HBC2 consists of the periodic BC on u' and d' fields, and the anti-periodic BC on u, d, s fields. Note that $\Lambda^{(5Q)}$ field consists of four quarks with original flavor and one anti-quark with additional flavor. N, Σ fields consist of three quarks with original flavor. \bar{K} and π fields consist of one quark with original flavor and one anti-quark with additional flavor. By repeating a similar consideration, we convince ourselves that $\Lambda^{(5Q)}$ is subject to the periodic BC, whereas N, Σ, \bar{K} and π

are subject to the anti-periodic BC. In this way, HBC2 can raise the thresholds both for $N\bar{K}$ and $\Sigma\pi$ while keeping a possible compact 5Q $\Lambda(1405)$ state unaffected.

For precision measurements, we use anisotropic lattice QCD, which has 4 times finer mesh along the temporal direction than the spatial directions as $a_s/a_t = 4$.^{1),7)} We employ the standard Wilson gauge action at $\beta = 5.75$ and $O(a)$ improved (clover) quark action with $\kappa = 0.1210(0.010)0.1240$, which covers roughly the quark mass region of $m_s \leq m_{u,d} \leq 2m_s$. $\kappa_s = 0.1240$ is fixed for s quark, while $0.1210 \leq \kappa \leq 0.1240$ is used for chiral extrapolation for u and d quark masses. The lattice spacing is determined with the Sommer parameter $r_0^{-1} = 395$ MeV, which leads to the spatial lattice spacing of $a_s^{-1} = 1.10$ GeV ($a_s \simeq 0.18$ fm). We use the lattice size $12^3 \times 128$. The spatial extension is $L \simeq 2.2$ fm. We use totally 2000 gauge field configurations. Along the temporal direction, we impose Dirichlet boundary condition on $t = 0$ plane. We employ a Gaussian smeared source with the Gaussian size $\rho \simeq 0.4$ fm, which is located on $t = t_0 \equiv 64$ plane. We utilise the time-reversal and charge conjugation symmetries to effectively double the statistics.

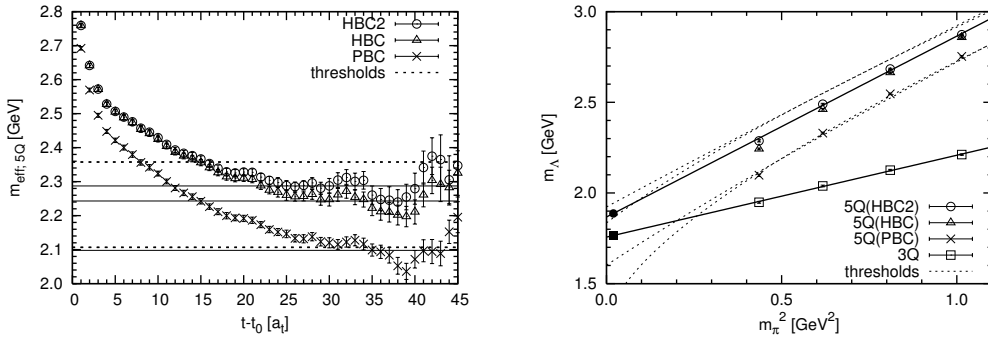


Fig. 1. The effective mass plot and the chiral extrapolation of the 5Q states which have same quantum number as the $\Lambda(1405)$. Two types of the hybrid boundary condition (HBC, HBC2) are applied to raise the threshold.

Fig. 1(left) shows the 5Q effective mass plot for HBC2(circle), HBC(triangle) and PBC(cross) for $\kappa = \kappa_s = 0.1240$ case. The lower dotted line denotes $N\bar{K}$ and $\Sigma\pi$ thresholds for PBC. HBC raises $N\bar{K}$ threshold, which is denoted by the upper dotted line. HBC2 raises also $\Sigma\pi$ threshold, which is denoted also by the upper dotted line. Note that, due to the flavor SU(3) symmetry, the (raised) $N\bar{K}$ threshold coincides with the (raised) $\Sigma\pi$ threshold. We see that a plateau is located at $t - t_0 \in [29, 45]$ for PBC data. It coincides with the PBC thresholds for $N\bar{K}$ and $\Sigma\pi$. For HBC and HBC2, the plateaux are raised above by about 200 MeV, which are located at $t \in [26, 45]$. It follows from these observations that the plateau for PBC data does not correspond to a compact 5Q state, but a scattering state. While the plateau for HBC2 is located below the raised thresholds by small amount of ~ 60 MeV, this is most probably not an indication of the existence of a compact 5Q states, considering the energy shift caused by remaining interaction in the finite volume. To clarify which is the case, further investigation is necessary. Since HBC does not affect $\Sigma\pi$ threshold, the fact that the plateau for HBC data is raised by about 200 MeV implies that our $N\bar{K}$ -type interpolating field has only a small overlap with $\Sigma\pi$ state. The

results of single exponential fits in these plateau regions are shown by solid lines.

Fig. 1(right) shows the results of linear chiral extrapolations. 5Q data for HBC2, HBC, and PBC are denoted with circle, triangle, and cross, respectively. The result of the 3Q data is also shown by square. Dotted curves denote the raised and un-raised thresholds for $N\bar{K}$ and $\Sigma\pi$. The solid lines denote the results of the chiral extrapolations for 5Q data with HBC2 and 3Q data. Note that the 5Q result $m_{5Q} = 1.887(9)$ GeV appears about 120 MeV above the 3Q result $m_{3Q} = 1.765(8)$ GeV, which is located about 360 MeV above the empirical value. This 5Q state is too heavy to be identified with the empirical $\Lambda(1405)$, which implies that $\Lambda(1405)$ is not a pure 5Q state.

To conclude, we have studied the 5Q picture of $\Lambda(1405)$ by using quenched anisotropic lattice QCD. By taking an advantage of quenched QCD, we have attempted to obtain a key to investigate a possible exotic structure of $\Lambda(1405)$. We have compared the results of the 5Q interpolating field with the result of the 3Q interpolating field, and have found that the 5Q results appear about 120 MeV above the 3Q result. Note that the 3Q result already overestimate the empirical value 1405 MeV by about 360 MeV. This has implied that $\Lambda(1405)$ is not a 5Q dominant state. The 5Q picture alone does not provide a solution to the significant overestimate of the mass of $\Lambda(1405)$ in quenched QCD. *Since $\Lambda(1405)$ is neither a pure 3Q state nor a pure 5Q state, we present an interesting possibility that $\Lambda(1405)$ is a mixed state of 3Q and 5Q states.* (Note that, the energy is generally reduced, if one seeks for a solution in a larger space.) The chiral quark effect may be also interesting.³⁾ Although quenched QCD with 3Q interpolating field leads to an imperfect overlap with intermediate 5Q states, 5Q contribution increases as the smaller quark mass region is approached. Of course, the annihilation diagram may be important, which we have neglected in our calculation to save the computational time. Since $\Lambda(1405)$ is such an interesting hadron, which may provide us with a possible exotic structure, we will keep studying on this interesting target from every aspect.

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